Variable time step smoothing filter

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Quick smoothing

- Camera movement
- Analog input smoothing
- Etc.
Naive algorithm

Fixed time step algorithm

```c
float c = 0.5f;
v2 = c * v1 + (1.0f-c) * vt;
```
Naive algorithm

Fixed time step algorithm formula

\[ v_2 = v_1 c + v_t (1 - c) \]

- \( v_1 \) is the current value.
- \( v_2 \) is the value next frame.
- \( v_t \) is the target value we are approaching.
- Smoothing response determined by \( c \).
- Note that \( \Delta t \) is not used.
Naive algorithm

Naive algorithm plot

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Variable $\Delta t$ filter
Variable time step algorithm

- Naive algorithm calculates exponential decay in fixed steps
- We need a function to calculate exponential decay with the same decay rate

**Exponential decay**

\[
e^{-\lambda \Delta t_f} = c
\]

\[
\ln \left( e^{-\lambda \Delta t_f} \right) = \ln (c)
\]

\[
-\lambda \Delta t_f = \ln (c)
\]

\[
-\lambda = \frac{\ln (c)}{\Delta t_f}
\]

Note: \( \Delta t_f \) is the delta time for the fixed frame rate algorithm
### Variable time step algorithm

\[ v_2 = v_1 \left( e^{-\lambda \Delta t} \right) + v_t \left( 1 - e^{-\lambda \Delta t} \right) \]

### Fixed time step algorithm

\[ v_2 = v_1 c + v_t (1 - c) \]
Exponential decay plot
Comparison

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Uses

- Smooth everything that can be linearly interpolated.
- Real-time camera smoothing without splines.
- Filtering without latency or converting to frequency domain.
- Low-pass filters with infinite impulse response.
- High-pass filters / edge detection.
- Simulating capacitors / RC circuit.
- Simulating radioactive decay.
- Motion blur.
Properties

- No memory buffers needed, just 1 sample.
- No latency as we don’t use future values.
- Infinite impulse response.
- Input set can be filtered by touching each element only once.
- Not separable, so not very usable for 2D blurring.
Low pass filter

\[ V_{in} \rightarrow R \rightarrow C \rightarrow V_{out} \]
Low pass filter

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Variable $\Delta t$ filter
Low pass filter

Capacitor discharge voltage

\[ V_C = V_0 e^{-\frac{t}{RC}} \]

RC low-pass filter cutoff frequency

\[ f_c = \frac{1}{2\pi RC} \]
Low pass filter

Capacitor discharge voltage

\[ V_C = V_0 e^{\frac{-t}{RC}} \]

Our RC

\[ e^{-\lambda \Delta t} = c \]
\[ e^{\frac{-\Delta t}{\lambda - 1}} = c \]
\[ RC = \lambda^{-1} \]
Low pass filter

**RC low-pass filter cutoff frequency**

\[ f_c = \frac{1}{2\pi RC} \]

**Our cutoff**

\[ f_c = \frac{1}{2\pi \lambda^{-1}} \]

\[ f_c = \frac{\lambda}{2\pi} \]