Variable time step smoothing filter

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Quick smoothing

- Camera movement
- Analog input smoothing
- Etc.

Naive algorithm

Fixed time step algorithm

```
float c = 0.5f;
v2 = c * v1 + (1.0f-c) * vt;
```

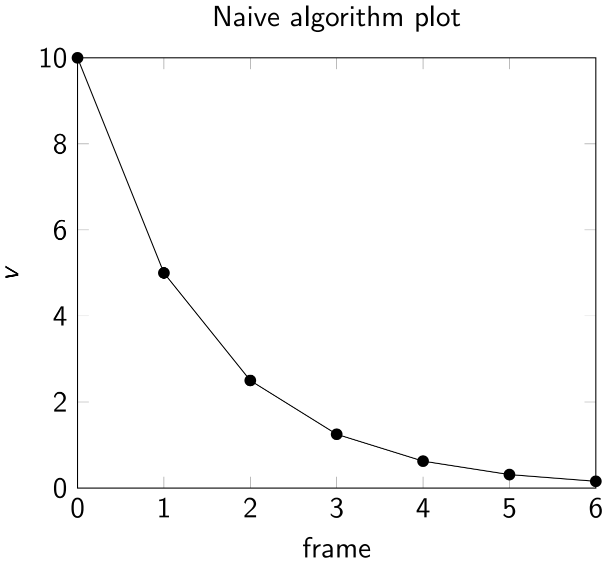
Naive algorithm

Fixed time step algorithm formula

$$v_2 = v_1 c + v_t(1 - c)$$

- v_1 is the current value.
- v_2 is the value next frame.
- v_t is the target value we are approaching.
- Smoothing response determined by c.
- Note that Δt is not used.

Naive algorithm



Variable time step algorithm

- Naive algorithm calculates exponential decay in fixed steps
- We need a function to calculate exponential decay with the same decay rate

Exponential decay

$$e^{-\lambda \Delta t_f} = c$$

$$\ln\left(e^{-\lambda \Delta t_f}\right) = \ln(c)$$

$$-\lambda \Delta t_f = \ln(c)$$

$$-\lambda = \frac{\ln(c)}{\Delta t_f}$$

Note: Δt_f is the delta time for the fixed frame rate algorithm

Variable time step algorithm

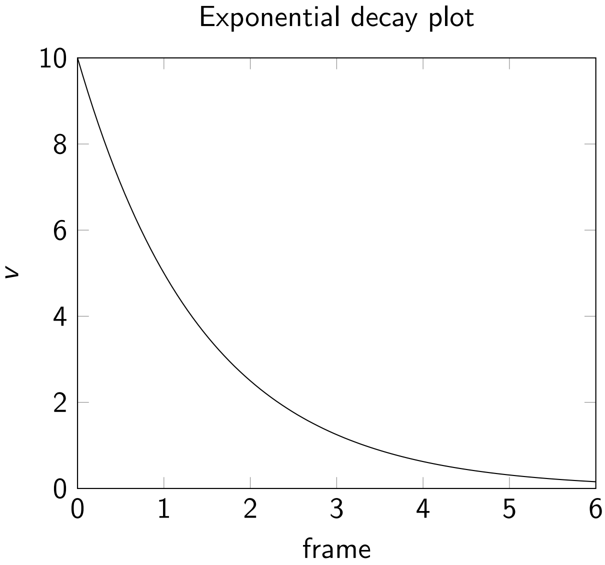
Variable time step algorithm

$$v_2 = v_1 \left(e^{-\lambda \Delta t}\right) + v_t \left(1 - e^{-\lambda \Delta t}\right)$$

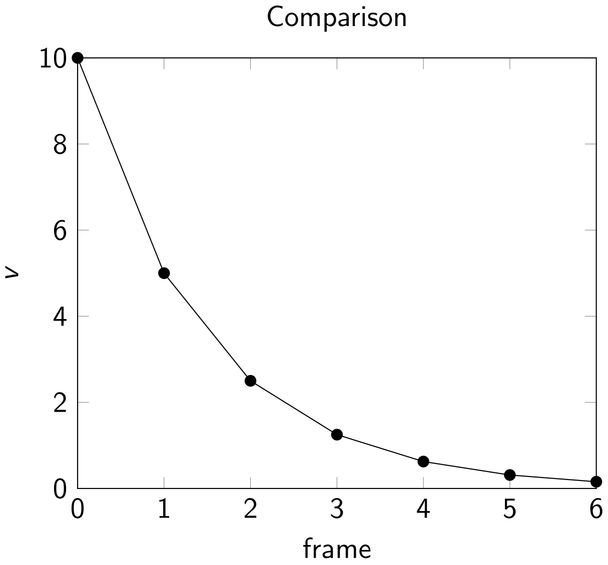
Fixed time step algorithm

$$v_2 = v_1 c + v_t(1 - c)$$

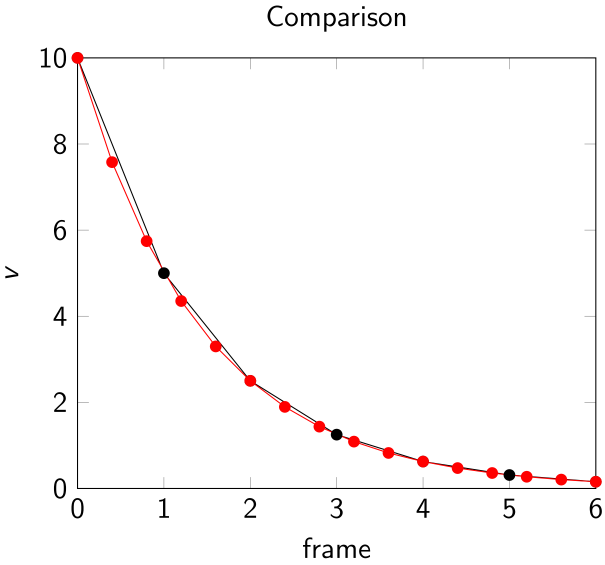
Exponential decay plot



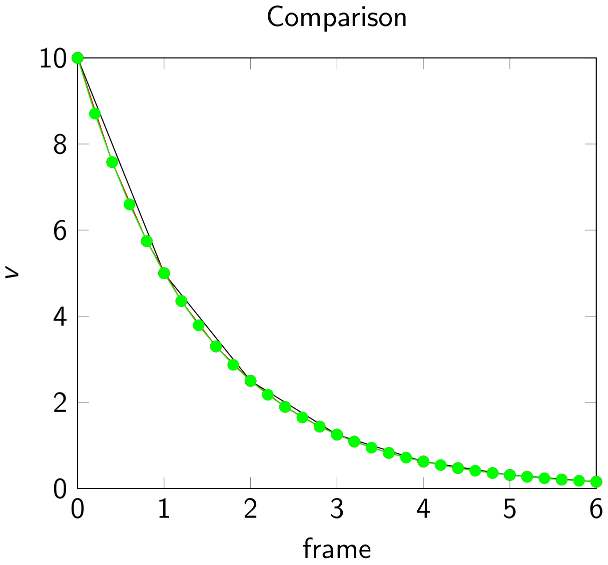
Comparison



Comparison



Comparison



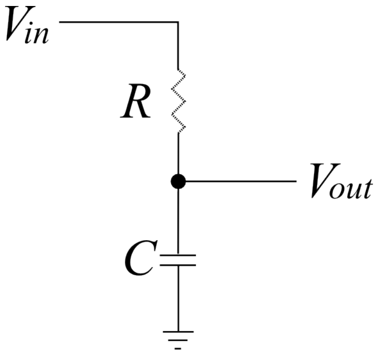
Uses

- Smooth everything that can be linearly interpolated.
- Real-time camera smoothing without splines.
- Filtering without latency or converting to frequency domain.
- Low-pass filters with infinite impulse response.
- High-pass filters / edge detection.
- Simulating capacitors / RC circuit.
- Simulating radioactive decay.
- Motion blur.

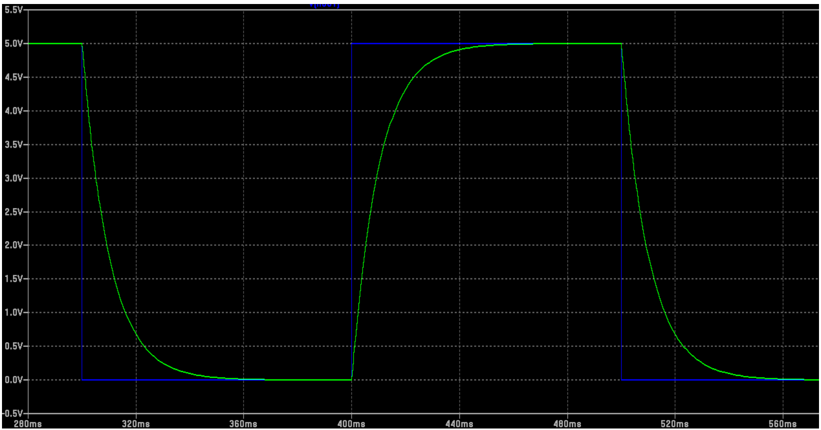
Properties

- No memory buffers needed, just 1 sample.
- No latency as we don't use future values.
- Infinite impulse response.
- Input set can be filtered by touching each element only once.
- Not separable, so not very usable for 2D blurring.

Low pass filter



Low pass filter



Low pass filter

Capacitor discharge voltage

$$V_C = V_0 e^{\frac{-t}{RC}}$$

RC low-pass filter cutoff frequency

$$f_c = \frac{1}{2\pi RC}$$

Low pass filter

Capacitor discharge voltage

$$V_C = V_0 e^{\frac{-t}{RC}}$$

Our RC

$$e^{-\lambda \Delta t} = c$$

$$e^{\frac{-\Delta t}{\lambda^{-1}}} = c$$

$$RC = \lambda^{-1}$$

Low pass filter

RC low-pass filter cutoff frequency

$$f_c = \frac{1}{2\pi RC}$$

Our cutoff

$$f_c = \frac{1}{2\pi\lambda^{-1}}$$

$$f_c = \frac{\lambda}{2\pi}$$